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by

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Abstract

A two-country, two-period model of international migration with heterogeneous agents highlights microeconomic foundations for examining the interrelation between brain drain, brain gain and whether human capital formation is undertaken at home or abroad. *Ex ante* choices regarding where to study depend on abilities, relative qualities of university systems, sunk educational investments, government grants, and endogenously determined, individual foreign employment probabilities. Self-selection critically defines an inherently wide-range of conceivably positive or negative net welfare effects. The optimal design of alternative educational grant schemes, aimed at enhancing the source country's welfare, also depends on the heterogeneity of abilities and associated informational assumptions.

***JEL classification codes*: F22, D82, I25, I28, J24**

***Key words*: human capital formation, brain gain, brain drain, international migration, sunk costs, educational grants, self-selection, asymmetric information**

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Section I: Introduction

Spawned by the noteworthy contribution of Bhagwati and Hamada (1974), the potentially adverse impact of the migration of skilled workers on a home country's growth and welfare has long been a key research concern. Nonetheless, early investigations also recognized conceivable advantageous effects for source countries, due to possible remittances and to temporary mobility, whereby migrants return from abroad with enhanced skills due to foreign job training. Another potentially important source of such brain gain, which is independent from return migration was identified by Mountford (1997) and Stark, Helmenstein and Prskawetz (1997), in what Schiff (2006) has termed the "new brain drain literature". Specifically, although migration can generate a loss of domestic talent, it can also prompt an upsurge in the overall educational level of a home country, as a result of higher propensities to invest in human capital. Attractive foreign labour market conditions offer heightened incentives for domestic workers to strive to attain higher qualification levels, whether or not they ultimately find jobs abroad, thereby fostering, *ceteris paribus*, increases in average productivity levels at home.[1] Yet, until recently, relatively little attention has been paid to the question of whether distinctive brain drain and gain effects may arise, depending on the extent to which educational investments take place either in home and/or host countries. Notably, pronounced international differences in educational quality and policies suggest a need to analyse explicitly individuals' arbitrage decisions regarding where optimal investments in human capital formation should take place. Such a focus is particularly warranted in light of the perceived high stakes and associated policy concerns arising from the increased international mobility of skilled workers.[2]

The potentially critical role of international educational choices on subsequent professional mobility, which has also been considered by both Rosenzweig (2008) and Docquier and Rapoport (2012), is particularly germane for motivating the modelling framework proposed in the current research. More specifically, Rosenzweig suggests two crucial limitations of existing approaches to the analysis of brain drain and gain. A first remark is that the potential impact of the "'risk' of emigrating" for "domestically-

[1] While macroeconomic frameworks with representative agents, such as in Vidal (1998) and in Stark and Zakharenko (2012), have examined the interrelation between international migration and domestic human capital formation, most existing approaches consider microeconomic decisions for a set of homogeneous individuals, as in the work of Stark, Helmenstein and Prskawetz (1997).

[2] See, for example, Leipziger (2008), OECD (2008), Solimano (2008), as well as Docquier and Rapoport (2012) for comprehensive surveys, which examine evidence regarding how the interface between globalization and brain drain/gain effects can impact the economic interests of source countries, depending on their levels of economic development.

educated tertiary educated person(s)" is *de facto* quite minimal. A second suggestion is that "the literature ignores the endogeneity of the emigration probability", while arguing that, in fact, "the choice of the location of tertiary education significantly affects the probability that the person can emigrate."[3](p. 61) In this regard, Docquier and Rapoport observed that "…workers trained at destination enjoy higher wages and employment rates than workers trained in their countries of origin…" (p. 693).

Nonetheless, much of the existing migration literature has left largely unexplored the extent to which brain drain/gain effects and the optimal design of government educational policies are conditioned by the heterogeneity of students' abilities, associated self-selection mechanisms and issues of informational asymmetries.[4] The present research responds to this lacuna by proposing a two-country, heterogeneous agent model, which offers a new theoretical paradigm for understanding the nexus between students' initial locational choices regarding human capital formation, differences in national labour market conditions, and international migration. The analysis underscores how net economic welfare in a home country can be impacted by an inherently wide-range of conceivable positive or negative categories of brain drain and gain effects, while also focusing on the optimal design of domestically financed educational grants (in a developing country).

Unique contributions result from an endogenous specification of the probabilities of foreign employment (brain drain) and return mobility (brain gain), which critically depend on heterogeneity of abilities. Such a formulation introduces a new dimension to associated calculations of economic welfare, since evaluations of such migration need to be quality-weighted and do not just reflect numbers of migrants. Crucially also, the heterogeneity of agents generates both ex ante and ex post self-selection mechanisms, along with potential issues of asymmetric information between public authorities and individuals. Such considerations, neglected in frameworks not incorporating sources of agent-specific heterogeneity, condition the extent of migration flows, as well as the efficacy and optimal

[3] While the analytical framework proposed by Rosenzweig does not allow for differences in individual abilities, his empirical findings are consistent with a number of the modeling assumptions which are subsequently invoked here. Notably, he reports evidence that students are motivated by foreign studies in order to obtain employment in a host country and that quality differences in university systems also appear to trigger the decision to study abroad.

[4] Stark, Helmenstein, and Prskawetz (1998) propose a scenario where each potential migrant faces identical probabilities of finding a job abroad, while in Stark (2004) there is a minimum threshold level of qualification. Mountford (1997) and Beine, Docquier, and Rapoport (2001, 2008)) investigate models where an individual's decision is of a binary form – whether to undertake education, or not - while the probability of finding foreign employment is exogenous. Hence, migrants are randomly selected. In contrast, Chiswick (1999) provides for self-selection by two categories of potential migrants individuals, such that the rate of return is greater for higher-ability individuals.

design of educational policies aimed at promoting national welfare.[5] **The heterogeneity of abilities also accentuates the potential effects of differences in the quality of international educational offerings on the post-educational productivity of workers, relative to a more standard framework where individuals are assumed to have equal talents. Heterogeneity typically generates potential threshold levels determining where individuals will chose to be educated, along with how government educational policies need to be tailored to such decisions.**

The analysis of how the impact of educational policies and systems on international migration can depend on agent heterogeneity has also been explored recently by Lange (2013) and Demange and Fenge (2010). A central concern is with the efficacy of different government educational strategies, including tuition cost policies and the quality of educational offerings, aimed at enhancing a country's public finances or national welfare. In this regard Lange (2013) has proposed a model in which students are heterogeneous in terms of their preferences for staying on abroad to work, following education in a host country, rather than in their abilities, as proposed here. The focus is on the optimal determination of tuition fees from the perspective of a developed host country. Demange and Fenge (2010) have developed a model of international student mobility in a two-country gaming framework. As in the analysis proposed here, the educational choices of students are heterogeneous in terms of their abilities. In light of cost-benefit evaluations, countries compete to attract capable students, who seek to invest in higher quality educational offerings. Nonetheless, there is also a critical difference, relative to the modelling framework formulated here, since the probability that an individual will return home is exogenously specified.

The rest of this paper is structured as follows. In Section 2 the basic modelling analysis starts with a sub-model of ex ante individual choice, regarding whether to undertake human capital formation at home or abroad. An individual's underlying ability determines known productivity gains from studying abroad, along with expected probabilities of subsequently obtaining foreign market employment at higher wages. The initial focus is on characterizing the ex post net impact of brain drain and brain gain on domestic economic welfare, which involves potential tradeoffs between productivity and social investment costs. Section 3 then presents

[5] The insight, formulated by Melitz (2003), that exporting firms are self-selected in terms of their productivity levels has spawned a reformulation of a significant part of the international trade literature. Yet, models of international migration, based on heterogeneous agents, are much less prevalent. Recent contributions in this regard include Krieger and Lange (2010), Lange (2013), Demange and Fenge (2010), and Demange, Fenge, and Uebelmesser (2014).

some comparative static results, relating to the welfare effects of changing key model parameters. A critical insight is that, in general, the associated net welfare effects can be either positive or negative. In Section 4 the relative welfare implications of alternative educational grant schemes, providing subsidies for foreign studies, are examined under alternative informational assumptions. A concluding section briefly summarizes certain salient findings, while identifying directions for further inquiry.

Section II: Basic Modelling Framework

Essential features of the proposed conceptual framework include the following:

1) **Individuals, from a home country, choose whether to undertake studies at home or abroad, where the latter entail an incrementally higher sunk investment cost. Foreign studies are understood to generate greater improvements in labour-market productivity, as compared with levels achievable through domestic human capital formation, where, crucially, the extent of realizable gains depends on underlying abilities across a heterogeneous population of individuals. If subsequently offered foreign employment, students opt to stay abroad because of higher wages, thereby generating brain drain. However, if individuals are unable to find suitable foreign employment, they still enjoy heightened productivity levels and wages, when returning home, as compared with workers who have not studied abroad. This generates brain gain.[6]**
2) **When modelling an individual's choice of whether to study abroad or stay at home, a crucial variable is the probability of being hired in the foreign labour market. Contrary to other models in which this probability is exogenous and identical for all graduates, it is assumed here to be a function of each individual's attainable level of qualification or, alternatively, productivity, where the latter depends on both individuals' abilities and the quality of educational institutions at home and abroad. As a consequence, migrants are "favourably self-selected" to use the terminology of Chiswick (1999).**
3) **The criterion chosen to assess brain drain/brain gain effects is the net impact on national welfare in the home country. This is represented, in a static framework, in terms of the change in domestic value-added resulting from foreign studies and eventual migration. The associated**

[6] There are also certain similarities between the general human capital formation-migration framework and the model of Kwok and Leland (1982), but their scenario does not include a brain gain effect.

welfare calculation depends, in turn, on the consequences for the home country's level of productivity, as well as the additional social costs resulting from eventual educational investments abroad. It is assumed there are no remittances.[7]

4) **Since foreign studies enhance productivity and thereby potentially lead to beneficial welfare effects, public authorities in the home country may seek, under certain conditions, to encourage foreign studies by subsidizing the candidates through alternative grant schemes, subject to a given overall budgetary constraint.**[8] **The welfare implications of three alternative grant policies are initially compared under an initial assumption that the public authorities have full information regarding students' underlying abilities. In particular, the impact of unconditional grants are compared to conditional grants, wherein students have the either the obligation to return to their home country, or the option of not doing so, provided their grants are repaid. Finally, certain implications of asymmetric information regarding the underlying abilities of grant recipients are explored.**

II. A. Sub-Model of Individual Investment in Human Capital Formation and International Migration

The initial focus is on the human capital investment decisions, in a first period, by heterogeneous individuals, who decide whether to pursue further studies at home, or abroad. Both their specific abilities and where they undertake further studies determine prospects for achieving enhanced productivity at the end of the period. Within a two-country setting, individuals, who initially choose to study at home, know that their job prospects, in a second period, will be confined to a lower-wage domestic market. In contrast, the pursuit of foreign studies offers prospects of higher productivity gains due to a conjectured superior quality of the foreign educational system. Individuals face a critical arbitrage, since there is an ex

[7] It is relatively straightforward to modify the proposed modelling framework, in order to allow for remittances, which would partially offset the negative welfare effects of brain drain. While such an extension potentially impacts quantitative results, it does not modify the essential qualitative insights summarized in subsequent propositions.

[8] Grants financing studies abroad are widely used, and can often be characterized in terms of the different grant categories considered here. For example, China, Thailand and Singapore offer conditional grants, which, while largely attributed in the sciences, require a return period of home country employment. For many developing countries, the associated financial commitments are considerable both in relative and absolute terms. For instance, in the case of Senegal a report of the Coordination of Technical and Financial Partnerships in Education for Senegal (2013) indicates that 37% of all higher-education grant aid to students in 2011 was for foreign studies, while the associated value of such funding amounted to almost 14.5 million dollars. In the case of the Republic of the Congo, the web journal, "Star du Congo" (April 7, 2014), reported that 32.6% of university-grant applications for the 2013-14 academic year were for studying abroad.

ante trade-off between improved employment prospects and higher sunk costs. While the pursuit of foreign studies offers higher salaries, individuals are initially uncertain regarding whether, or not, they will be subsequently hired abroad.

More specifically, out of an overall population of N individuals in the domestic country, N_0 represents the number of domestic individuals who remain at home for both their education and work, while N* is the total number of persons who choose to undertake foreign studies and, subsequently, work either at home, or abroad. Thus, there are two distinct sub-populations of N*, corresponding to the phenomena of "brain gain" and "brain drain". In particular, N_1^* designates the number of domestic individuals who chose to get educated abroad and subsequently work in the foreign country, while N_1 corresponds to the number of domestic individuals who are educated abroad, but then return home to work. In sum, whereas higher values of N_1^* generate greater brain drain, increases in N_1 results in more brain gain.

The overall domestic population of N individuals are understood to differ in terms of their innate intellectual and work capacities, which for the k*th* individual, can be denoted as a_k. The attainable productivity levels for students depend not only on their underlying abilities and the quality of the initial educational system in the home country, designated as q_1, but also on where further educational investments are to be undertaken at home, or abroad. In this regard, it is hypothesized that the quality of the domestic higher educational system, Q_1, is inferior to that offered in the foreign country, Q_2. Hence, there is an educational production function that determines how investments of fixed amounts of time in a particular educational system map individuals' capacities into their effective qualifications or productivity levels, e_k, such that $e_k=f(a_k, q_1, Q_j)$, where j=1,2.[9] This functional relation results in a range of attainable productivity levels, measured on a scale between, e_0 and e_2. For subsequent simplicity, a value of e_0 is used as a numeraire to designate a unique level of productivity for all of the N_0 domestically educated workers, regardless of their inherent capacities. However, workers trained abroad, N_1^* or N_1 , enjoy higher final productivity levels, which are distributed, according to their innate abilities, on an interval from e_1 to e_2, as represented by a density function, h(e).

[9] More generally, the value of the k*th* individual's human capital investments depends on the amount of time spent on education, the quality of university educational systems and his/her ability. While the analysis here only provides for individuals undertaking higher educational studies in a single period and in only one country, it could be extended to allow for students spending different periods of time, either at home or abroad. The returns from educational investments could depend, then, on the specific stage of university, or earlier, studies, as well as country-specific differences in educational quality, which could be highly variable according to educational levels.

While offering the prospect of higher productivity gains, the decision to undertake foreign studies is understood to entail higher educational costs, I*, compared to the costs of pursuing further education in the home country, I_0. Hence, in the absence of educational grants, students will be willing to incur this difference between the foreign and domestic educational costs, designated as $i = I^* - I_0$, provided two conditions are met. First, the expected higher wage returns arising from enhanced productivity gains must offset the net cost differential for paying for higher quality studies abroad. Second, financial markets are assumed to be perfect. Accordingly, students can readily borrow against their expected future earnings, in order to finance the immediate sunk costs of educational investments, inclusive of financing charges.

Individuals' ex ante willingness to incur sunk costs of educational investments is clearly impacted by anticipations regarding the labour market conditions they face after graduating - both at home and abroad. The latter are reflected both by hiring prospects and wage differentials between the two countries. In the proposed framework, individuals who have been educated abroad have the ex post option of seeking employment abroad at a higher wage, than in their home market. For the overall population of N* workers, who are educated abroad, each individual, designated by the subscript k, faces a probability, p_k , of finding qualified employment abroad. This probability plays a crucial role in the analysis, as it delineates "brain drain" from "brain gain" effects. Notably, two extreme cases, where p_k equals either 1, or 0, correspond, respectively, to pure brain drain or brain gain effects. For more intermediate value of p_k , both the phenomena of brain drain and brain gain will arise, respectively, in the proportions p_k and $1 – p_k$ across the overall population N*.

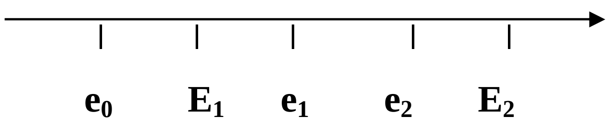
Nonetheless, in the proposed model, the probability of finding employment abroad varies across the heterogeneous population of individuals, since it depends *endogenously* on their expected levels of productivity, which, in turn, are related to underlying abilities and educational choices. More specifically, each of the p_k values is taken to depend linearly on the level of the effective qualifications realized by the k*th* individual, e_k, relative to a threshold value, E_1 , reflecting a minimum standard in the foreign labour market, and negatively on the range of skill requirements, E_2 - E_1, such that:

$$\textbf{(1.)} \quad p_k = p(e_k) = \frac{(e_k - E_1)}{(E_2 - E_1)}$$

Figure 1 offers a representative illustration of the assumed distribution of effective qualification levels for domestic individuals, in relation to the skill requirements of the foreign labour market. Intermediate values for the parameters E_1 and E_2 are assumed, where these threshold values, respectively, preclude or guarantee foreign market employment. Thus, in the proposed model, each foreign-trained, domestic-origin, student faces a non-zero probability of finding employment abroad. A previously indicated simplification is that individuals, who chose to remain at home for their education, are unable to work abroad.[10]

Figure 1

The Assumed Structure of Skill Levels Attainable at Home or Abroad, Relative to Foreign Labour Market Requirements



The parameters, E_1 and E_2, can be understood to reflect foreign labour market conditions, as well as educational and employment policies. For example, employment standards abroad can be influenced by the overall quality of the foreign educational system (including that of pre-university studies), as well as by technology-driven, labour-demand requirements. Different combinations of these parameter values can also be interpreted to represent alternative immigration policies, since higher values could correspond to more restricted labour market access, while depending on the skill intensities of available jobs in the foreign country. Moreover, lower values of E_2 could, ceteris paribus, represent a situation of relative shortages for specific categories of highly skilled workers. Furthermore, lower (higher) values of both of these foreign market parameters can be interpreted as facilitating (hindering) the immigration of foreign skilled workers.

Following their studies, foreign-trained domestic students have an incentive to seek employment abroad due to the higher foreign salaries, w^*, for skilled jobs, whereas returning students can only earn a lower reservation wage in their home country, equal to w_1.[11] For tractability, both of these salaries are assumed to be unique values, which are independent of

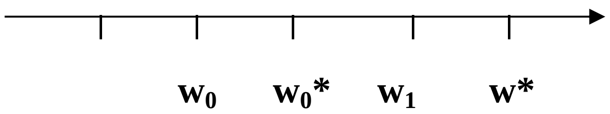
[10] Eventual rationale for this assumption include an inadequate relative quality, or high-degree of specificity, of the domestic educational system, positive professional network effects on employment abroad arising from foreign studies, and/or restrictive visa and related immigration policies, favouring students trained in a host country.

[11] A fixed exchange rate equal to unity is assumed.

students' effective qualification levels achieved through their pursuit of studies abroad. Furthermore, it is assumed that the reservation wage, facing returning students, is higher than both the remuneration offered to wholly domestically trained workers, w_0 , and the foreign wage, which they can earn in less skilled jobs abroad, w_0^*.[12] Within this proposed framework, students, who are unsuccessful in finding appropriate skilled work in the foreign country, will return home.[13] While the wage rates are taken to be exogenous, the subsequent analysis will consider comparative static changes in their values, reflecting the relative attractiveness of labour market conditions internationally. Figure 2 summarizes, then, the overall international structure of wages, depending on job locations and educational backgrounds

Figure 2

The Structure of International Wages According to Job Location and Educational Background



The *ex ante*, optimal educational choice, for the representative k*th* student involves a trade-off, which can be formulated in terms of an arbitrage condition. Specifically, the net returns from studying and working at home, with lower overall effective qualifications, need to be compared to expected higher wage earnings, arising from enhanced productivity due to foreign studies, albeit at a greater investment cost. The expected wage remuneration involves a probability-weighted average of wages for more skilled workers in the foreign and domestic markets. Accordingly, a representative student will decide to study in the foreign country if:

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[12] This feature of the model could be modified to allow for an analysis of issues relating to brain waste, were w_0^* to be higher than w_1.

[13] Of course, other factors, such as cultural affinities, as well as personal and family considerations, could offset the locational incentives of these ex post wage differentials between the two countries. Such additional factors generate an additional ex post self-selection problem, which can be modeled in terms of agent-specific complementary or substitutable assets and associated sunk costs. These considerations amplify the degree of heterogeneity characterizing individuals' decisions, thereby accounting for distinctive evolutionary sub-populations of agents, following, in this instance, the revelation of news regarding job offers. For example, ceteris paribus, if students have a preference to return home, there will be an increase in brain gain effects, relative to those identified in the subsequent analysis. Analogously, when foreign students marry persons from a host country, they may be prepared to stay abroad even without optimal employment conditions, thereby forming part of brain waste.

$$\textbf{(2.)}\quad p_k w^* + (1-p_k)w_1 - I^* > w_0 - I_0$$

Hence, the k*th* individual will decide to study abroad if his/her individual probability of being hired abroad, p_k is higher than a critical probability value, $\bar{p}$. This probability is assumed to depend on a student's, potentially private, information regarding his/her future productivity level, e_k. More specifically, the interrelation between this critical probability value, $\bar{p}$, and the prevailing international wage rates and educational costs are given by:

$$\textbf{(3.)}\quad \bar{p} = \frac{i-(w_1-w_0)}{w^*-w_1} \quad \textbf{if } \frac{i-(w_1-w_0)}{w^*-w_1} \in [0,1]$$

$$\bar{p} = 0 \qquad \textbf{if } \frac{i-(w_1-w_0)}{w^*-w_1} < 0 \textbf{, that is if } i < w_1 - w_0$$

$$\bar{p} = 1 \qquad \textbf{if } \frac{i-(w_1-w_0)}{w^*-w_1} > 1 \textbf{, that is if } i > w^* - w_0$$

From (1.), it follows that the productivity level corresponding to $\bar{p}$ is: $\tilde{e} = (E_2 - E_1)\bar{p} + E_1$ However, $\tilde{e}$ does not necessarily belong to the segment of productivity levels attainable from foreign studies, [e_1 , e_2], so that the actual productivity threshold is $\bar{e}$ such that

$$\textbf{(4.)}\quad \bar{e} = \tilde{e} = (E_2 - E_1)\bar{p} + E_1 \quad \textbf{if } \tilde{e} \in [e_1, e_2],$$

$$\bar{e} = e_1 \qquad \textbf{if } \tilde{e} < e_1,$$

$$\bar{e} = e_2 \qquad \textbf{if } \tilde{e} > e_2.$$

The foregoing specifications permit a characterization of the distinctive populations of students, depending on both ex ante educational choices and the ex post employment prospects. In particular, out of the overall population of N students, the number of students choosing to remain at home is given by $N_0 = N\int_{e_1}^{\bar{e}} h(e)de$, whereas the complementary set of individuals studying abroad amounts to $N - N_0 = N\int_{\bar{e}}^{e_2} h(e)de$. The latter group can be sub-divided into two sets of individuals, corresponding to brain drain and brain gain effects, represented, respectively, by $N_1^* = N\int_{\bar{e}}^{e_2} p(e)h(e)de$ and $N_1 = N\int_{\bar{e}}^{e_2} [1-p(e)]h(e)de$. Whereas the foregoing analysis assumes exogeneous wages, an analogous decomposition of the overall population of students also applies where salaries depend positively on productivity levels. Provided the

salary differential between the two markets, $w^*(e) - w_1(e)$, is a non-decreasing function of productivity, the expected wage returns from opting to study abroad remain an increasing function of e. Consequently, there also exists a threshold level $\bar{e}$ determining whether, or not, individuals will undertake foreign studies. As a result, the subsequently reported findings in this section are robust to such an alternative formulation.[14]

Production, or value-added at home is taken to be characterized by a linear function, reflecting a proportional relation to productivity. Thus, if individuals were not able to study abroad, national output would be $Y_0 = e_0$ N, which constitutes an essential benchmark under educational autarchy, since then workers are only trained domestically. The contribution to national production generated by the foreign-educated individuals returning home corresponds, then, to $Y_1 = N\int_{\bar{e}}^{e_2} e[1-p(e)]h(e)de$. Hence, $Y_1 - e_0 N_1$ constitutes the incremental increase in national income resulting from brain gain.

II. B. Economic Welfare in the Home Country

The evaluation of brain drain/brain gain effects in the existing literature is based on assessments of the impact of migration on a variety of specific economic objectives, which, however, do not include an explicit social welfare function. Notably, migration is shown to influence the growth rate of the home economy, as in Beine, Docquier, and Rapoport (2001), the average educational level, as highlighted by Stark et al. (1997, 1998) and Lien and Wang (2005), average productivity in Mountford (1997), as well as the wages of non-migrants in Stark (2004).[15]

A distinctive feature of the proposed analysis is the explicit consideration of how brain drain and brain gain effects, linked to international human capital formation, impact domestic social welfare, relative to the level under autarchy, Y_0. In this perspective, changes in welfare generated by international educational and employment mobility can be viewed in terms of a cost-benefit analysis relating to changes in

[14] Nonetheless, the exogeneity assumption invoked here, regarding wages and abilities, considerably enhances the tractability of mathematical derivations.

1While an extensive number of empirical studies have assessed different dimensions of the potential impact of brain drain and gain, there remains a lack of consensus regarding the size of conjectured positive effects of migration upon levels of education, welfare and/or growth. Notably, Beine, Docquier, and Rapoport (2001, 2008) find that the proportion of migrants must be low for such effects to be apparent. According to Schiff (2006), preliminary studies by the World Bank show no positive impact, while Groizard and Llull (2006) indicate a similar finding.

national aggregate productivity and the net costs of educational expenditures borne by the home country. Nonetheless, such a focus will abstract from potential distributional issues concerning relative returns in terms of wages, firms' profits, as well as transfers between the domestic government and the private sector.

The net return in terms of productivity generated by a student returning home amounts to $e - e_0$, while the net opportunity cost of that person's education equals I*- c, where c corresponds to the social cost of educating an individual domestically. For the case of a student remaining abroad, the corresponding effects involve a loss of national productivity, e_0 , minus a gain amounting to c, since there is no need to incur domestic educational costs. In this regard, it should be noted that future salary gains are used to pay off the costs of a foreign education, I*, so that there is no associated social cost at home. In sum, the net cost-benefit evaluation for the brain gain resulting from a representative student returning home amounts to $e - e_0 - (I^* - c)$, whereas, for an individual entailing a brain drain effect, the corresponding value equals $-(e_0 - c)$.[16]

More explicitly, the overall change in domestic welfare is determined by the brain drain and gain effects corresponding to individuals who study abroad, whose productivity levels are comprised between $\bar{e}$ and e_2. This amounts to a variation in social welfare equal to:

$$(5.)\quad \Delta W = N\int_{\bar{e}}^{e_2} \langle [1-p(e)][e-e_0-(I^*-c)] - p(e)(e_0-c)\rangle h(e)de$$

This can be expressed equivalently as :

$$\Delta W = N\int_{\bar{e}}^{e_2} \langle [1-p(e)](e-I^*) - (e_0-c)\rangle h(e)de$$

As a simplification, the subsequent analysis will assume an uniform distribution of attainable productivity levels, comprised between lower and higher bounds of e_1 and e_2. In light of such a specification, the overall net change in welfare becomes:

$$(6.)\quad \Delta W = \frac{N}{e_2-e_1}\int_{\bar{e}}^{e_2} \varphi(e)\, de = N\frac{\Phi(e_2)-\Phi(\bar{e})}{e_2-e_1}$$

[16]The educational costs for society of training students, prior to their deciding to study abroad and, subsequently, working permanently there, could also, arguably, be considered to negatively impact domestic social welfare. There would then be an additional term, negatively impacting domestic welfare, as a result of brain drain. On the other hand, the proposed specification of the social welfare function does not allow for the positive impact of remittances, which would depend on the value of w*, along with different propensities characterizing individuals' decisions to transfer funds back home. Incorporating such effects entails relatively straightforward extensions of the basic modelling framework proposed here.

Here, the function φ constitutes the overall net welfare effect for each attainable productivity level. When the overall social opportunity cost for a student who ultimately works abroad is identified as $\delta = e_0 - c$, then, in light of equation (1.), the expression for φ equals:

$\varphi(e) = \frac{E_2 - e}{E_2 - E_1}(e - I^*) - \delta$, **while Φ in equation (6.) represents the primitive of the function φ.**

As shown by equation (6.), the incremental change in domestic welfare is a function of all the parameters of the model. To summarize, it depends on:

- **e_0: the productivity of less-skilled domestically-trained workers;**
- **e_1 and e_2: the two extreme values defining the range of enhanced productivity levels for foreign-educated workers;**
- **E_1 and E_2: parameters reflecting foreign market skill requirements and labour market access conditions, which impact the probability of finding work abroad;**
- **$\bar{e}$: the threshold value of productivity, which decides whether an individual chooses to study abroad, which, in turn, is impacted by among other factors, the wages of skilled workers employed abroad, w^*, those for skilled workers employed at home, w_1, and the wages of unskilled workers at home, w_0;[17]**
- **I^*: the cost of foreign studies;**
- **I_0 and c: the cost of studies at home per student, borne, respectively, by each individual and by society.**

The expression for the primitive function in equation (6.), Φ, which critically defines the extent of the change in domestic welfare, is of the third degree in e. The underlying reason for such a functional form is the second degree form for the integrand, $\varphi(e)$, in equation (6.), which represents the expected increase in net welfare for a representative individual. This expression involves a trade-off between the expected increase in productivity realized through brain gain, $e(1-p(e)) - e_0$, and the expected net social cost of educating a student abroad, $(1-p(e))I^* - c$. Since the former quadratic term in e assumes low values for either relatively low or high productivity values, the values of the integrand are initially negative, then positive (for sufficiently low i) and finally negative, as representative productivity levels for different individuals increase.

[17] As shown by considering equations 3 and 4.

As illustrated in Figure 3, the general form of the primitive function Φ may first show a minimum, for e = $\hat{e}_1$, and then a maximum for e = $\hat{e}_2$. Of course, these extrema exist if and only if the equation φ(e) = 0 admits real roots, which corresponds to the following condition:

$$\textbf{(7.)}\quad \boldsymbol{\delta < \frac{(E_2 - I^*)^2}{4(E_2 - E_1)}}\,.$$

When I* and δ, which jointly determine the social cost of a foreign education, are too high, Φ is always a decreasing function of e. Consequently, the change in domestic welfare, ΔW, is always negative, so that the brain drain effect dominates that of brain gain. The value for which Φ has a minimum, $\hat{e}_1$, is relevant only if the latter is greater than E_1. Calculations show that the associated condition is simply:

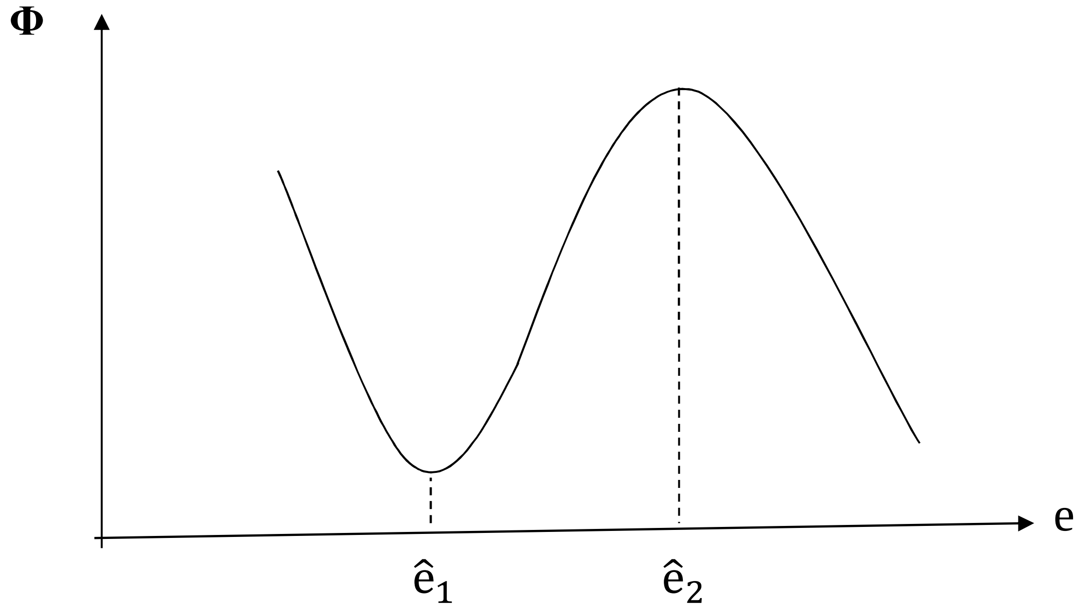
$$\textbf{(8.)}\quad E_1 < I^* + \delta\,.$$

In the rest of the paper, it is assumed that conditions (7.) and (8.) are always satisfied.

It should also be noted that the integrand φ has a maximum. Specifically, the function φ takes on low values when e is itself low, since in that case the individual productivity gains are too weak to compensate for the cost of foreign studies. For high values of e, when few students come back home, there is a loss of productivity for society as a whole, so the value of φ is also low. This maximum occurs for e = $(E_2 + I^*)/2$, which corresponds to an inflection point for the curve representing function Φ, such that the marginal increase in social welfare associated with a marginal decrease of the productivity threshold $\bar{e}$ is the highest.

Figure 3

Representation of the Functional Form for the Primitive Function, Φ, which Determines the Overall Change in Domestic Welfare



Section III: An Analysis of the Effects on Economic Welfare of Changes in Key Model Parameters

III.1 The Interrelation between Threshold Productivity Levels and Changes in Welfare

The initial focus here is on the welfare implications of the critical value of $\bar{e}$, which reflects the threshold productivity level for which a representative individual chooses to study abroad. The value of $\bar{e}$ in relation to $\hat{e}_2$ is potentially of key importance. Note again that $\bar{e}$ is a function of the critical threshold probability, $\bar{p}$, triggering foreign study, as well as of the foreign labour market productivity requirements, E_1 and E_2, since $\bar{e} = (E_2 - E_1)\bar{p} + E_1$. In turn, $\hat{e}_2$ is a function of E_1, E_2, and the social opportunity cost of foreign studies, δ. Hence, it follows that $\bar{e} > \hat{e}_2$ for $\bar{p} > p_{lim}$, where $p_{lim} = \frac{\hat{e}_2 - E_1}{E_2 - E_1}$. Furthermore, $\hat{e}_2 < E_2$, since $\varphi(E_2) < 0$, so that the value of p_{lim} is always inferior to one; while it could be negative, this would mean that $E_1 > \hat{e}_2$. However, this corresponds to a relatively uninteresting case,

where domestic welfare always declines, as a result no individuals study abroad. For more relevant scenarios, there is an actual probability threshold beyond which $\bar{e} > \hat{e}_2$. It can also easily be seen that when $\bar{e}$ increases and $e_2 > \hat{e}_1$, ΔW has a maximum for $\bar{e} = \hat{e}_1$. Thus, if initially $\bar{e} < \hat{e}_1$, a marginal increase in $\bar{e}$ promotes welfare. However, if $\bar{e} > \hat{e}_1$, an increase in $\bar{e}$ reduces the number of people who study abroad, thereby reducing welfare. An examination of Figure 3 and a comparison of the values taken by the function Φ for $\bar{e}$ and e_2, leads then to the following:

Proposition 1

When the threshold productivity level, $\bar{e}$, determining whether individuals will study abroad, and the upper limit on the associated level of enhanced productivity, e_2, both belong to the interval $[\hat{e}_1, \hat{e}_2]$, the change in welfare resulting from studying abroad, ΔW, is positive. Hence, the welfare improvement from brain gain dominates the loss due to brain drain.

In contrast, there are three cases where foreign studies generate a loss of welfare. Notably,

a) when $\bar{e}$ and e_2 are both very low, the return to foreign education, in terms of increased productivity, is weak and does not compensate for its social costs, even if many individuals study abroad and return home to work ;
b) when $\bar{e}$ and e_2 are both very high, few individuals leave to study abroad, but most of these will readily find a job abroad, resulting in a dominance of the brain drain effect;
c) when $\bar{e}$ is low and e_2 is high, there is an accumulation of the foregoing effects a) and b). Notably, many individuals study abroad, thereby generating high additional educational investment costs, but only those with lower-productivity gains return home.

In sum, the welfare implications of comparative static changes in productivity levels, $\bar{e}$ and e_2, are inherently ambiguous.

III.2 The Configuration of Wages and Associated Welfare Effects

The influence of wages on domestic welfare works through changes in the critical values for $\bar{p}$ and $\bar{e}$. As can be expected, higher wages for domestically trained workers create, ceteris paribus, a disincentive to studying abroad, so when w_0 increases, both $\bar{p}$ and $\bar{e}$ increase. However, when the potential job market returns to foreign studies w^* or w_1 increase, the incentives to studying abroad are increased, so that $\bar{p}$ and $\bar{e}$ are

lowered. The associated consequences for domestic welfare stem from the preceding analysis of the influence of $\bar{e}$. Specifically, if $\bar{e}$ is not very low (inferior to $\hat{e}_1$), an increase (decrease) in wages for foreign-trained (domestic-trained) workers, decreases (increases) $\bar{e}$, thereby enhancing welfare.

<u>III.3 Welfare Implications of Changes in the Relative Productivity Gains from Education at Home and Abroad</u>

A heightened efficiency for domestically-trained workers, e_0, which corresponds to an improved domestic educational system, reduces the net impact on welfare of brain drain and brain gain, by increasing the opportunity cost of undertaking foreign studies. In contrast, an increase in the lower limit of the enhanced efficiency level attained via foreign studies, e_1, raises the returns to a foreign education, and induces a larger proportion of the population to study abroad. The effect of a variation in e_2 is more complex to assess. By widening the span of productivity values, an increase of e_2, ceteris paribus, has a negative influence upon ΔW. If $e_2 > \hat{e}_2$, $\Phi(e_2)$ also decreases, so that the overall effect is also negative. However, if e_2 belongs to the interval $[\hat{e}_1 , \hat{e}_2]$, $\Phi(e_2)$ increases and the net effect is indeterminate. More specifically, the formula for the derivative of ΔW is:

$$\text{(9.)} \quad \frac{d\Delta W}{de_2} = \frac{N}{(e_2 - e_1)^2} \{\Phi(\bar{e}) - [\Phi(e_2) - (e_2 - e_1)\, \varphi(e_2)]\} .$$

It can be seen that, if $\bar{e} < \hat{e}_2$, so that ΔW may be positive, then the foregoing expression is negative for $e_2 = \hat{e}_2$. Consequently, the change in the domestic country's welfare has a maximum for some value of e_2 (also inferior to $\hat{e}_2$). In light of the foregoing analysis, the following holds:

<u>Proposition 2</u>

The change in the domestic country's welfare, ΔW, is an increasing function of the level of e_2 , the maximal level of enhanced productivity achievable by undertaking studies abroad, provided e_2 remains under a critical level. Beyond this threshold, ΔW decreases with e_2 . Thus, too much of an improvement in human capital, or, alternatively, relative excellence in the foreign institutions, generates a dominant effect of brain drain.

The associated critical value of e_2 is increasing with the threshold productivity level determining whether students go abroad, $\bar{e}$, and decreasing with the lower limit of the value of enhanced productivity, e_1 .

Finally, if both e_1 and e_2 increase with a constant span between the two values, ΔW has a maximum for $e_2 = \hat{e}_2$. Such a change could result from improvements in the quality of individuals' initial education in the home country and/or in higher educational standards abroad. In that case, from the perspective of domestic welfare, there is also an optimal level of relative efficiency in the foreign educative system. Any increase of foreign educational efficiency above this level will diminish home national welfare, thereby constituting a form of "beggar-thy-neighbour" policy.

III.4 Changes in the Sunk Cost Differential for Studying Abroad

As the additional sunk costs associated with foreign studies, i, increase, the threshold probability of finding a job abroad increases, as does the corresponding threshold productivity level, $\bar{e}$. Furthermore, if this increase in i comes from an increase in I*, the integrand function φ decreases. As a consequence, so long as $\bar{e} \in [\hat{e}_1, \hat{e}_2]$, an increase in the incremental costs of studies abroad, i, reduces the home country's welfare. In contrast, for low values of $\bar{e}$ ($\bar{e} < \hat{e}_1$), an increase in i could possibly be beneficial. In such a scenario there is initially an excessive flight of students abroad, since, for a representative student, the productivity gains from a foreign education are high, whereas the additional costs, i, are low.

III.5 Alternative Immigrant Employment Policies in the Foreign Country

The relative ease of access to the foreign labour market is captured here by alternative values for the labour market requirement parameters, E_1 and E_2. Ceteris paribus, for higher values of either parameter it is more difficult for a domestic-origin, but foreign-trained, job-searcher with a given qualification level to be employed abroad. More specifically, when either E_1, or E_2 increase, $\bar{e}$ increases, but $p(e_k)$ decreases for any value of e_k. Crucially, there are two offsetting effects. On the one hand, fewer individuals leave to become educated abroad, but, on the other hand, a greater fraction of graduated students come back home. Thus, the total pool of foreign trained students from the domestic country is reduced. This means that the overall exposure of the domestic country to welfare changes, arising from either brain drain or brain gain, decreases. However, the relative proportion of foreign-trained students generating a brain gain increases as a result of the more restrictive job filtering environment in the foreign country. Consequently, the net effect on domestic welfare is potentially ambiguous.

As demonstrated in Appendix 1, the following summary conclusion applies:

Proposition 3

Restrictions limiting entry by foreign-trained students to the host country's labour market increase home national welfare, provided the following conditions hold:

a. the cost of undertaking foreign studies is high;
b. the maximum achievable productivity level, e_2, is relatively low; and
c. relatively few individuals undertake studies abroad
(i.e. $\overline{p}$ is near 1).

In contrast, if the foregoing conditions are not satisfied, then less favourable foreign labour market conditions result in a negative impact on domestic welfare.

Section IV: A Comparative Analysis of the Domestic Welfare Implications of Alternative Educational Grant Schemes

The focus in this section is on the optimal design of educational grants, aimed at enhancing a domestic country's welfare by facilitating foreign study for different categories of students. Since alternative subsidy programs can change the incentives to study abroad, they potentially impact the balance between brain drain and brain gain, which, in turn, determines the net changes in domestic welfare.

Two distinct analytical exercises are proposed depending on contrasting scenarios regarding the extent of a government's knowledge of students' underlying abilities. Initially, it will be assumed that both the government and students themselves have perfect information regarding individuals' abilities. In a second scenario, it is assumed instead that the government, unlike the students themselves, has no knowledge regarding individuals' capabilities.

In the first scenario of perfect information the government can discriminate ex ante between individuals when allocating grants according to three different grant schemes.[18] The first of these entails unconditional grants (designated as UC), which are awarded without any constraints on students regarding either financial repayments, or subsequent employment

[18] For simplicity, the analysis here will focus on grants entailing uniform payments for all recipients, rather than discriminating between individuals in terms of the proposed value of the grants.

choices. Under a second scheme, labeled as optional return (OR), any grant recipient may opt to work in the foreign country, but must then pay back the full value of the grant. Finally, under a third scheme, identified as compulsory return (CR), students commit to returning home to work, even if they could otherwise have been employed abroad at a higher wage.

IV.1 Unconditional Grants

When an omniscient government has full information regarding the underlying ability of all students, grants will be allocated as a function of a candidate's ability, or, equivalently, in light of the associated expected productivity gain. Of course, individuals whose productivity is superior to the standard threshold $\bar{e}$ will never be grant beneficiaries, since there is no need for any additional financial incentive to undertake foreign studies. Under this system, all individuals, whose productivity levels are comprised between a designated level, $\bar{e}_b$, and $\bar{e}$, may be candidates for a grant. The lower productivity limit for the grant recipients, $\bar{e}_b$, is endogenously determined by the per capita value of the foreign educational subsidies, S. This amount also depends, in turn, on the government's overall educational budget constraint. Nonetheless, it is not necessarily optimal to give a grant to the brightest students (whose productivity is relatively close to the threshold value of $\bar{e}$), since such students are less likely to return home to work. Hence, grants will be given, a priori, to students whose productivity belongs to a certain segment of productivity values, $[\bar{e}_b, \varepsilon_b]$, with $\varepsilon_b \leq \bar{e}$.

The new productivity threshold, $\bar{e}_b$, is determined by an ex ante, individual arbitrage condition such that:

$$\textbf{(10.)} \quad \overline{p_b} w^* + (1 - \bar{p}_b) w_1 - (I^* - S) = w_0 - I_0$$

where $\bar{p}_b = p(\bar{e}_b)$ indicates the threshold probability for grant beneficiaries. Then, there is a standard interrelation between such a probability value and the associated threshold productivity level, $\bar{e}_b$, such that: $\bar{e}_b$=($E_2 - E_1$)$\bar{p}_b$+ E_1. It is straightforward to see that,

$$\textbf{(10.)} \quad \bar{p} - \bar{p}_b = \frac{S}{w^* - w_1}$$

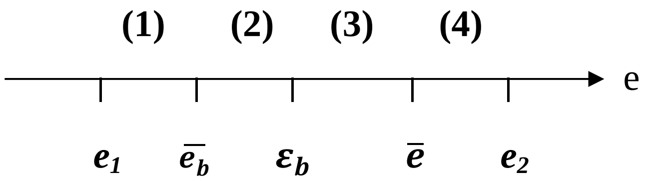
so that,

$$\textbf{(12.)} \quad \bar{e} - \bar{e}_b = \frac{S(E_2 - E_1)}{w^* - w_1}$$

As a result, for any given proposed values for the individual foreign study grants, as well as an overall budget for the government grant program, the subset of students actually going abroad is shown in Figure 4.

Figure 4

Structure of the population for unconditional grants



zone 1 corresponds to individuals for whom the proposed grant is not sufficient to convince them to go abroad
zone 2 corresponds to grant recipients who undertake foreign studies due to the grant
zone 3 corresponds to individuals who do not receive a grant and study at home
zone 4 corresponds to individuals, who while not receiving any grant, still undertake foreign studies

Now let F be the overall budget allocated for grants. The number of beneficiaries will be $\frac{N}{e_2-e_1}l$, where $l=\varepsilon_b-\bar{e}_b$ corresponds to the productivity interval of grant recipients. Hence, the government budget constraint may be written $\frac{N}{e_2-e_1}l\ S=F$, or, equivalently, in light of equation (12.),

$$\textbf{(13.)}\ (\varepsilon_b-\bar{e}_b)(\bar{e}-\bar{e}_b)=kF$$

where $k=\frac{E_2-E_1}{w^*-w_1}\frac{e_2-e_1}{N}$.

Under this scheme, the increase of welfare generated by allowing additional individuals to study abroad is:

$$(14.)\ \Delta W_b=\frac{N}{e_2-e_1}\int_{\bar{e}_b}^{\varepsilon_b}\varphi(e)\,de$$

The maximum value of ΔW_b is reached for values of $\bar{e}_b$ and ε_b, such that $\bar{e}_b \geq \hat{e}_1$ and $\varepsilon_b \leq \hat{e}_2$. This corresponds to an upper sloping segment of the

primitive function, Φ, which determines the overall change in domestic welfare, as depicted in Figure 3. Otherwise, were $\bar{e}_b$ to be inferior to $\hat{e}_1$, it would be possible to increase ΔW_b, for a given ε_b, by increasing $\bar{e}_b$, while still reducing the educational budgetary expenditures. Analogously, if ε_b were to be superior to $\hat{e}_2$, it would be possible to increase ΔW_b by decreasing ε_b with $\bar{e}_b$ constant, while reducing again budgetary expenditures. Nonetheless, a maximum value for ΔW_b with ε_b = $\hat{e}_2$ may not be feasible since, by construction, it must be the case that $\varepsilon_b \leq \bar{e}$.

If it is assumed that $\bar{e} \geq \hat{e}_2$, then ε_b can reach $\hat{e}_2$, which is its unconstrained optimal value. Provided the level of available funds permits such a value for ε_b and $\bar{e}_b$, the optimum will then be $\varepsilon_b = \hat{e}_2$ and $\bar{e}_b = \hat{e}_1$, that is if $(\bar{e} - \hat{e}_1)\,(\hat{e}_2 - \hat{e}_1) \leq kF$. If the available public funds are relatively low, the budgetary constraint will be binding and the optimum value of ε_b will be inferior to $\hat{e}_2$. Yet, in any case the optimal value of ε_b will be strictly inferior to $\bar{e}$.

Instead, if it is assumed that $\bar{e} < \hat{e}_2$, the constraint $\varepsilon_b \leq \bar{e}$ may become binding. The optimum then corresponds to $\varepsilon_b = \bar{e}$ and $\bar{e}_b = \hat{e}_1$, conditional on the level of funds being large enough, as expressed by the constraint $kF \geq (\bar{e} - \hat{e}_1)^2$. However, it is shown in Appendix 2 that, provided that the budgetary constraint is binding, there exists a threshold productivity value, $\bar{e}_{min}$, and associated intervals of values for $\bar{e}$ and F, where, respectively, $\bar{e} \in [\bar{e}_{min}\,,\ \hat{e}_2]$ and $F \in [F_1, F_2]$, such that ΔW_b has a maximum for some value of $\bar{e}_b$, where the corresponding ε_b is inferior to $\bar{e}$. When these conditions are not satisfied, ΔW_b is always an increasing function of $\bar{e}_b$. Since ε_b is always inferior or equal to $\bar{e}$, the optimum is associated with the maximum value for $\bar{e}_b$, which corresponds to $\varepsilon_b = \bar{e}$, and is given by $\bar{e}_b = \bar{e} - \sqrt{kF}$. This means that the segment of productivity values $[\bar{e}_b, \bar{e}]$ is fully covered by the allocation of the grants, while the value of $\bar{e}_b$ depends on the level of the available funds.

The foregoing analysis can be summarized, as follows:

<u>Proposition 4</u>

When a government has full knowledge regarding students' abilities, a scheme of unconditional grants (UC), which targets individuals who are not among the most qualified (relative to $\bar{e}$), can be optimal under specific conditions.

In particular, the welfare effects of such unrestricted grants are more favourable when either of the following combination of productivity and budgetary conditions apply: i. $\bar{e} > \hat{e}_2$, or ii. $\bar{e} \epsilon [\bar{e}_{min}, \hat{e}_2]$ and $F \in [F_1, F_2]$.

Thus, the foregoing proposition highlights an apparent paradox in the funding of unconditional grants for studies abroad, which are aimed at promoting national welfare. Specifically, in the presence of potential brain drain due to more attractive working conditions abroad, it may, somewhat paradoxically, be preferable to offer grants to somewhat less qualified individuals in an interval of potential productivity levels lying between $\bar{e}_b$ and ε_b. The existence of such threshold levels is analogous to a phenomenon often encountered in the optimal design of public subsidies. These can entail boundary conditions for subsets of the population, ranked in terms of income.

IV.2 Conditional Grants

As previously indicated, two systems of conditional grants are envisaged. These correspond to the cases of either optional (OR) or compulsory return (CR) to work in the home country.

A. The Case of Optional Return

In this scenario a student, after graduation, will either find a job abroad and reimburse the educational grant money, or decide to return home. Under such a scheme, the net expected income of a student who receives a conditional grant, amounting to S_{OR}, but with an optional return is:

$$p(e)[w^* - S_{OR}] + [1 - p(e)]w_1 - (I^* - S_{OR}) =$$
$$p(e)w^* + [1 - p(e)]w_1 - I^* + [1 - p(e)]S_{OR}$$

In light of the last term in this expression, it follows that the net expected gain for such a grant recipient is always superior to the expected income without a grant, regardless of a student's productivity.

As a consequence, the overall pool of grant beneficiaries will be defined by a productivity threshold level, $\bar{e}_{OR}$, which is inferior to the standard threshold $\bar{e}$, and where $\bar{e}$ and $\bar{e}_{OR}$ are, respectively, defined by:

$$\textbf{(15.)} \quad p(\bar{e})w^* + [1 - p(\bar{e})]w_1 - I^* = w_0 - I_0$$

$$\textbf{(16.)} \quad p(\bar{e}_{OR})[w^* - S_{OR}] + [1 - p(\bar{e}_{OR})]w_1 - (I^* - S_{OR}) = w_0 - I_0$$

By comparing equations (15.) and (16.), it can be seen that:

$$(17.)\quad S_{OR} = (w^* - w_1)\frac{p(\bar{e}) - p(\bar{e}_{OR})}{1 - p(\bar{e}_{OR})} = (w^* - w_1)\frac{\bar{e} - \bar{e}_{OR}}{E_2 - \bar{e}_{OR}}$$

Note that the inequality $S_{OR} < w^* - w_1$ always holds. As a consequence, if a student, who has accepted the grant, is proposed a foreign job, that individual will always accept it rather than return home. The grant will be given to students whose productivity levels are comprised between $\bar{e}_{OR}$ and ε_{OR}. Since for each productivity value e, only a proportion of $1 - p(e)$ students come back, the budgetary constraint is given by:

$$(18.)\, \frac{N}{e_2 - e_1}\lambda_{OR} S_{OR} = F$$

where

$$(19.)\, \lambda_{OR} = \int_{\bar{e}_{OR}}^{\varepsilon_{OR}} [1 - p(e)]\, de$$

By combining (17.), (18.) and (19.), this constraint becomes:

$$\frac{\bar{\mathrm{e}} - \bar{e}_{OR}}{\mathrm{E}_2 - \bar{e}_{OR}} \int_{\bar{e}_{OR}}^{\varepsilon_{OR}} (E_2 - e) de = kF$$

The change in social welfare corresponds then to:

$$(20.)\, \Delta W_{OR} = \frac{N}{e_2 - e_1} \int_{\bar{e}_{OR}}^{\varepsilon_{OR}} \varphi(e)\, de$$

B. The Case of Compulsory Return

Here, a grant recipient is obliged, following studies abroad, to return home and work in the domestic labor market. Two conditions must be met for a representative student k to accept such a grant, which amounts to S_{CR}. First, the earnings from returning to work at home, following studies abroad, must be superior to those associated with staying at home to both study and work. Hence, this condition corresponds to:

$$w_1 - (I^* - S_{CR}) \geq w_0 - I_0$$

Second, the expected net returns must also be superior to those for an individual, who chooses to study abroad without accepting such a grant with a compulsory return home:

$$w_1 - I^* + S_{CR} \geq p(e_k)w^* + [1 - p(e_k)]w_1 - I^*$$

This condition leads to $e_k \leq \tilde{e}$, where the threshold $\tilde{e}$ is an increasing function of S_{CR}. It follows that $\tilde{e} = \bar{e}$, when $S_{CR} = I^* - I_0 - (w_1 - w_0)$, which is the minimum value, such that the proposed grant would be accepted. For higher values of S_{CR}, the upper productivity limit $\tilde{e}$ will be superior to $\bar{e}$. Certain of these higher-ability students may opt to accept the grant, despite the binding commitment to return home, since they may prefer not to have to pay by themselves the full cost of foreign studies. Thus, in this instance there is a key analytic issue of whether it is preferable for the public authorities to offer grants exclusively to students who would not leave without a grant (i.e. whose productivity levels are lower than $\bar{e}$), or to also propose grants for brighter students, who would study abroad even in the absence of government subsidies.[19]

In the first sub-case of grants for less able students, the productivity interval, characterizing the recipients, spans an interval of length $l_{CR} = \bar{e} - \bar{e}_{CR}$, so that the budgetary constraint becomes:

$$\text{(21.)} \quad \frac{N}{e_2 - e_1} l_{CR} S_{CR} = F$$

Clearly, for a given amount of the grant, S_{CR}, this constraint determines the range of grant recipients, l_{CR}. The incremental welfare generated by such grants is specified then by:

$$\text{(22.)} \qquad \Delta W_{CR} = \frac{N}{e_2 - e_1} \int_{\bar{e}_{CR}}^{\bar{e}} (e - A)\, de$$

where A is the total social cost of sending a student abroad, such that $A = I^* + \delta$.

For the second sub-case, as shown in Appendix 3, it is optimal to give grants first to the brightest students, whose productivity goes by descending order from e_2 to ε_2, where $\varepsilon_2 \geq \bar{e}$. Yet, if the budgetary constraint is not binding, educational support will also be offered to less talented students, whose productivity levels lie between $\bar{e}$ and ε_1. Here, the amount of the grant is determined by the value of e_2: $S'_{CR} = p(e_2)(w^* - w_1)$, while the budget constraint then amounts to:

$$\text{(23.)} \qquad \frac{N}{e_2 - e_1} l'_{CR} S'_{CR} = F$$

[19] A potentially worthwhile extension of the analysis would be to consider the role of inter-temporal constraints on educational investment decisions. Income inequality can then play a critical role in influencing what subsets of students will chose to study abroad, when there are associated higher educational costs.

where $l'_{CR} = (e_2 - \varepsilon_2) + (\bar{e} - \varepsilon_1)$.

The additional welfare generated by such a grant scheme, includes a sub-component corresponding to students whose productivity levels are such that $e \geq \bar{e}$. The net welfare improvement for each of these students amounts to $e - A - \varphi(e) = p(e)(e - I^*)$, so that total incremental change in welfare is specified by:

$$\text{(24)} \quad \Delta W'_{CR} = \frac{N}{e_2 - e_1} \int_{\varepsilon_1}^{\bar{e}} (e - A)\, de + \frac{N}{e_2 - e_1} \int_{\varepsilon_2}^{e_2} p(e)(e - I^*)\, de$$

A priori, it is difficult to assert for which of these sub-cases the largest welfare improvements, ΔW_{CR} or $\Delta W'_{CR}$, arise. Nonetheless, when either the social opportunity cost of foreign studies, A, is high enough, or when $\bar{e}$ is relatively low, then ΔW_{CR} is constrained to be rather small. This is reflected by the expression for the change of welfare in equation (22.). Hence, it is better to start such a grant scheme by proposing financing to the brightest students.

The analysis will now turn to a more detailed comparison of the welfare gains potentially generated by each of the three overall grant systems.

IV.3 A Comparative Welfare Analysis for Different Grant Schemes

An initial comparison will be made between the welfare implications of unconditional grants and conditional grants, where in the latter instance returning to work in the home labor market is optional. As a partial simplification, this analysis will be limited to scenarios where the productivity levels $\bar{e}$ and e_2 are comprised between $\hat{e}_1$ and $\hat{e}_2$. Such a restriction avoids additional complications, which can arise when studying abroad without grants does not always generate enhanced social welfare. A principal finding can be summarized as follows:

Proposition 5

For the same overall budget, conditional grants with an optional return (OR) always generate greater increases in social welfare, relative to unconditional grants (UC), so that OR > UC.

The associated proof is provided in Appendix 4.1 .

The extension of this comparative analysis of grants schemes, to consider a scenario where returning home is compulsory (designated by CR),

entails further complications. As previously demonstrated in Section IV.2.B, there are two different expressions for the welfare benefits of such grants. However, for analytical simplicity the analysis here will be confined to a consideration of the limiting cases where the level of public funding for such grants with compulsory return is either very small, or very large. In these cases, it can be shown that:

<u>Proposition 6</u>

a) When available funds are very small, the system of grants with optional return results in the greatest improvement in national welfare, while conditional returns constitute the least favorable scheme. Accordingly, the national social preference ranking for the three schemes can be summarized by the following inequalities: OR > UC > CR.

b) In contrast, when available funds are very large, the system of grants with compulsory return is preferable to the other two schemes, so that: CR > OR > UC.

The associated proof is provided in Appendix 4.2.

<u>IV.4 Grants under Asymmetric Information</u>

The analysis will now consider implications of imperfect knowledge regarding individuals' abilities for assessing the optimality of different grant schemes. In an extreme scenario, where the government does not have any information regarding students' innate abilities, the welfare impact of public authorities awarding uniform grants for foreign studies can be explored. The same individual monetary subsidies are granted to a proportion of the overall population, independently of students' abilities. When the government faces a specific educational budget constraint, it can be demonstrated that welfare will always increase, provided the modelling parameters are such that the analysis applies to the upward sloping segment of the primitive function, Φ. However, for certain other parameter values welfare may actually decrease. Furthermore, for $\hat{e}_1 < \bar{e} < e_2 < \hat{e}_2$, the optimal proportion of the population, α, which should receive such awards in order to enhance welfare, equals 1. Hence, the overall grant budget is divided evenly across the entire population.

More generally, an apparent limitation of any grant scheme with imperfect information is that individuals with potential productivity levels

above the threshold value of $\bar{e}$ will receive subsidies which constitute a deadweight social loss, since they would have studied abroad anyway in the absence of such financing. This reasoning can be extended to consider the welfare implications of such grants resulting from comparative static changes in the overall quality of the basic educational system in the home country, q_1 . When the initial educational system is of a higher quality, the interval between e_1 and e_2 shifts to the right, so that the relative position of the threshold level $\bar{e}$ is lowered within that segment of productivity values. Accordingly, as information becomes more imperfect, educational grants generate heightened social welfare inefficiencies. This potentially immiserizing feature of educational grants under uncertainty can also be formulated as follows:

Proposition 7

When public authorities do not have any information regarding students' abilities, and an unconditional grant scheme for foreign studies increases domestic welfare, the extent of such a welfare improvement is inversely related to the quality of a home country's initial educational system, q_1 .

Such a lessened efficacy of unconditional grants is due to a greater deadweight social loss, since educational improvements lead to a relatively higher proportion of grant recipients, who would have studying abroad anyway in the absence of such grants. This effect aggravates potential inefficiencies generated by asymmetric information. Thus, under such informational conditions a home country faces an apparent trade-off between enhancing its economic welfare via improvements in initial levels of educational attainment and through grant schemes aimed at offering incentives to study abroad. Consequently, the extent to which information is incomplete can condition the optimal design of grant schemes, depending on countries' standards of living and the quality of their educational systems.

Asymmetric information has apparent implications for the earlier identified paradox characterizing the optimal design of unconditional grants, which were proposed for sub-portions of the population, who are not necessarily among the most qualified. Now, it is no longer feasible to distinguish students according to their abilities. This analysis can also be extended to consider optional return grants under incomplete information. Specifically, when there are imperfections in the extent to which public authorities can identify individuals' abilities, there is potentially an incentive for students to distort their performance, in order to qualify for grants. Such incentive compatibility issues need to be taken into consideration when formulating optimal grant policies. Unlike unconditional and optional

return grant schemes, grants entailing a compulsory return effectively eliminate such strategic distortions. This is due to an auto-selection process, whereby only those candidates, whose anticipated net earnings will increase, accept such grants, thereby revealing their true abilities. Furthermore, with reference to Proposition 6b), it can be shown for the case of uniform grants that imperfect information results in an expanded set of instances where grants with compulsory returns dominate unconditional and optional return grants.

Section V: Conclusion

A distinctive goal of this research has been the formulation of a micro-founded heterogeneous-agent model of international migration and educational choice, thereby highlighting critical roles for self-selection mechanisms and informational asymmetries. Distinctive features of the analysis arise from the endogenous determination of human capital investment decisions and foreign employment prospects, which differ across individuals according to their abilities. This structure provides a framework for understanding the interrelation between international educational choices and employment prospects, along with aggregate assessments of the domestic welfare implications of brain drain and brain gain. Heterogeneity and associated informational issues also condition the optimal design of alternative educational grant schemes, aimed at increasing a source country's well-being.

Certain more specific insights from this research can be summarized, as follows. Crucially, the net welfare impact of foreign studies on international migration, in the absence of educational grants, is very difficult to assert, as a general proposition. Instead, the determination of such effects requires detailed examination of relatively complex interactions between an array of economic factors, which influence individuals' optimal educational investment decisions. A crucial methodological issue is the non-linearity of the welfare changes, reflecting associated brain drain and brain gain effects, in relation to the distribution of workers' productivity levels. Nevertheless, when the threshold minimum productivity value, determining whether individuals leave, and the maximum attainable level for the population of foreign-educated students are both relatively average, in comparison with foreign labour-market productivity requirements, the net welfare effect resulting from foreign human capital formation is positive, i.e. brain gain dominates brain drain. In this case, welfare is a decreasing function of the threshold probability of finding a job abroad, and, thereby, of the investment cost differential between foreign and domestic studies. Welfare is

also an increasing function of wages paid to foreign-educated skilled workers, in either the home, or foreign labour markets, and a decreasing function of wages paid to less-skilled domestic-trained workers at home. In contrast, either very low, or relatively large values for the fore-mentioned productivity parameters foreign studies are associated with detrimental welfare implications, since brain drain dominate brain gain effects.

The analysis has subsequently examined the optimal design and relative efficiency implications of three alternative measures, aimed at improving economic welfare in the home country by offering financial incentives to study abroad, under different informational assumptions. These grant schemes, consisting of unconditional grants, and conditional grants with, or without, obligations of returning home to work, are initially examined while assuming full information regarding the heterogeneous abilities. Somewhat paradoxically, it may be preferable to propose unconditional grants to a subset of somewhat less qualified individuals, in light of lessened vulnerability to brain drain effects. Yet, for a constant budget, unconditional grants are dominated by conditional grants with an optional return. Nonetheless, in comparison with these two initial grant schemes, the relative efficacy of grants with a compulsory return to work depend critically on the size of the budgets for funding such programs. In particular, compulsory grants are inferior (superior) to these other grants, when funding is relatively limited (generous). Finally, an extension of the analysis to consider grants under asymmetric information establishes that when an unconditional grant scheme improves domestic welfare, the extent of such an improvement is inversely related to the quality of a home country's initial educational system. The subsequent analysis also points to a strengthened rationale for using grants with a compulsory return, when public authorities face informational asymmetries.

Certain potentially fruitful directions for extending the analysis proposed here entail incorporating additional modelling features. These include admitting the possibility that domestically educated students can also seek employment on the foreign labour market.[20] A critical consideration would then be the differential probability of finding a foreign job, which depends on the gap between the productivity distributions for home and foreign-educated domestic workers, as well as the specificity of training to employment in different countries. The latter could be captured by iceberg style effects impacting the degree of convertibility of qualifications across labour markets. Clearly, a crucial consideration may be the extent to which the educational system in the home country enables particularly capable

[20] Such an extension is relatively straightforward within the present modelling framework, when there is a fixed probability of being hired abroad for domestically-trained individuals.

students to enhance substantially their productivity levels, or, in other words, the extent of educational elitism. A more detailed analysis of the interrelation between alternative educational policies in the home country and the extent of brain drain and gain could also consider the interrelation between the quality of education offered at different educational levels, the pricing of such studies and the extent of their subsidization – both at home and abroad. A basic presumption would be that there are potential welfare trade-offs between the budgetary expenses of improving national educational offerings and allocating funds for educating students abroad, which could depend on the associated net balances between brain drain and gain. A generalization of the present modelling framework could also permit an analysis of the strategic interactions arising from alternative educational budgetary and policy initiatives in both the home and foreign countries. Alternative scenarios relate to the extent to which both implicit and explicit government subsidies impact the migration of talent and associated implications for the international distribution of welfare. As in the current research, the dependency of such effects on the interrelation between tuition fees, underlying educational costs, and the overall quality of educational offerings across educational levels and systems internationally will potentially be defined by underlying issues of agent heterogeneity and the extent of informational asymmetries.

In light of well-known market failures for financing investments in human capital, initial income distributions could play a critical role in determining whether individuals are prepared to study abroad without government funding. Consequently, an additional policy option could be analysed by incorporating alternative hypotheses regarding income and asset distributions and introducing unconditional and/or conditional loans for less wealthy students. If educational loans specify that recipients must return home to work, they generate only brain gain, thereby enabling governments to counter issues of asymmetric information regarding their knowledge of individuals' underlying abilities, since more talented students would, ceteris paribus, tend to accept such loans. Finally, a dynamic modelling perspective, with alternative assumptions regarding sources of underlying individual heterogeneity, could highlight how alternative growth paths for the home economy depend on human capital investments in at home and/or abroad, eventual migration, and endogenous adjustments in wages. Nonetheless, there are apparent methodological challenges to extending the existing heterogeneous-agent framework.

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APPENDIX 1
Consequences of Alternative Employment Policies in the Foreign Country

The analysis here examines the effects of changing the foreign labour market requirement parameters, E_1 and E_2. The specific demonstration of Proposition 3 starts by considering a comparative static change in E_2, for a given value of E_1 : $\frac{d\Delta W}{dE_2} = \frac{N}{e_2 - e_1}\left[\int_{\bar{e}}^{e_2} \frac{\partial \varphi}{\partial E_2}(e)\, de - \varphi(\bar{e})\frac{\partial \bar{e}}{\partial E_2}\right]$ **. The foregoing expression contains two terms, which can be simplified as follows:** $\frac{\partial \varphi}{\partial E_2} = \frac{(e - I^*)(e - E_1)}{(E_2 - E_1)^2} = \bar{p}^2 \frac{(e - I^*)(e - E_1)}{(\bar{e} - E_1)^2}$ **and** $\frac{\partial \bar{e}}{\partial E_2} = \bar{p}$ **. As a result, one obtains:** $\frac{e_2 - e_1}{N}\frac{d\Delta W}{dE_2} = \bar{p}^2 \int_{\bar{e}}^{e_2} \frac{(e - I^*)(e - E_1)}{(\bar{e} - E_1)^2} de$ **-** $\bar{p}$ **$[(\bar{e} - I^*)(1 - \bar{p}) - \delta]$. By defining** $G(\bar{e}) = \bar{p}\int_{\bar{e}}^{e_2} \frac{(e - I^*)(e - E_1)}{(\bar{e} - E_1)^2} de$**, and re-expressing the term algebraically, it follows that**

$$\frac{1}{\bar{p}}\frac{e_2 - e_1}{N}\frac{d\Delta W}{dE_2} = G(\bar{e}) - [(\bar{e} - I^*)(1 - \bar{p}) - \delta].$$

Note that G is a positive decreasing function of $\bar{e}$ such that $G(e_2) = 0$, and φ is an increasing function of $\bar{e}$, for given $\bar{p}$. Consequently, if $\varphi(e_2) < 0$, that is if $e_2 < I^* + \frac{\delta}{1 - \bar{p}}$, $\frac{d\Delta W}{dE_2}$ is always positive $\forall \bar{e} < e_2$. Accordingly, the change in domestic welfare, ΔW, is always increasing with $\bar{e}$, and so also with E_2. However, if $e_2 > I^* + \frac{\delta}{1 - \bar{p}}$, there is a threshold value for $\bar{e}$ such that beyond this value, ΔW is decreasing when E_2 and $\bar{e}$ increase. Yet, this threshold value may be inferior to e_1 , in which case ΔW is always decreasing with E_2 . Furthermore, qualitatively similar results hold for an increase in E_1 , or for an increase in both E_1 and E_2, when, in the latter case, a constant span $E_2 - E_1$ is assumed.

APPENDIX 2
Unconditional Grants

A starting point for the analysis is the expression of the derivative of ΔW_b with respect to $\bar{e}_b$. Since the value of the parameter $\frac{N}{e_2 - e_1}$ does not matter here, it can be arbitrarily set equal to 1, in order to simplify the notation. Accordingly, the following expression applies:

$$\frac{d\Delta W_b}{d\bar{e}_b} = \varphi\,(\varepsilon_b)\frac{d\varepsilon_b}{d\bar{e}_b} - \varphi\,(\bar{e}_b).$$

Along the budget constraint, $\varepsilon_b = \frac{kF}{\bar{e} - \bar{e}_b} + \bar{e}_b$ and $\frac{d\varepsilon_b}{d\bar{e}_b} = \frac{kF}{(\bar{e} - \bar{e}_b)^2} + 1$.

It can be seen that $\frac{d\Delta W_b}{d\bar{e}_b}$ is positive for $\bar{e}_b = \hat{e}_1$ and negative for $\bar{e}_b = \hat{e}_2$. Of particular interest here is the case where $\bar{e} < \hat{e}_2$. The upper limit value for ε_b is, then, $\bar{e}$, which corresponds to $\bar{e}_b = \bar{e} - \sqrt{kF}$. If $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e})$ is negative, ΔW_b has a maximum for a value of ε_b strictly inferior to $\bar{e}$. On the contrary, if $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e})$ is still positive, it means that the optimum corresponds to the limit value $\varepsilon_b = \bar{e}$. For $\varepsilon_b = \bar{e}$, one has:

$$\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e}) = \varphi\,(\bar{e})\left[\frac{kF}{(\bar{e} - \bar{e}_b)^2} + 1\right] - \varphi\,(\bar{e}_b)$$

with $\bar{e}_b = \bar{e} - \sqrt{kF}$ and, thus, $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e}) = 2\,\varphi\,(\bar{e}) - \varphi\,(\bar{e} - \sqrt{kF})$. Given then that $\varphi(e)$ has a maximum for $e = E_2/2$, it follows that if $\bar{e} \leq E_2/2$, $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e})$ is certainly positive. Actually this is still true, provided $\bar{e} \leq \bar{e}_{min}$, where $\bar{e}_{min} \in [E_2/2, \hat{e}_2]$ is such that $\varphi\,(\bar{e}_{min}) = 1/2\,\varphi\,(E_2/2)$.

When $\bar{e} \in [\bar{e}_{min}, \hat{e}_2]$, the sign of $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e})$ depends on the value of F. More precisely, this derivative is negative when F belongs to an interval $[F^1, F^2]$, for which the limits are functions of $\bar{e}$, and solutions of the equation $2\,\varphi\,(\bar{e}) - \varphi\,(\bar{e} - \sqrt{kF}) = 0$. Furthermore, the higher the value of $\bar{e}$, the wider is the

interval, which has a maximum for $\bar{e} = \hat{e}_2$ **, corresponding to** $F^1 = 0$, $F^2 = k(\hat{e}_2 - \hat{e}_1)^2$**. This shows also that for** $\bar{e} \in [\bar{e}_{min}, \hat{e}_2]$ **, all F belonging to** $[F^1, F^2]$ **meet the condition** $F \leq k(\bar{e} - \hat{e}_1)^2$**, while this condition is satisfied as an equality only for** $\bar{e} = \hat{e}_2$ **, and** $F = F^2$ **.As a result, when** $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e}) < 0$**, the change in welfare,** ΔW_b **, has again an interior maximum for values of** $\bar{e}_b$ **and** ε_b **satisfying the equations** $\frac{d\Delta W_b}{d\bar{e}_b}(\bar{e}) = \varphi(\varepsilon_b)[\frac{kF}{(\bar{e} - \bar{e}_b)^2} + 1] - \varphi(\bar{e}_b) = 0$ **and** $k(\bar{e} - \bar{e}_b)(\varepsilon_b - \bar{e}_b) = F$ **.When F does not belong to the fore mentioned interval,** $\frac{d\Delta W_s}{d\bar{e}_s}(\bar{e}) \geq 0$**, so that the maximum value of** ΔW_s **corresponds to** $\bar{e}_b = \bar{e} - \sqrt{kF}$ **and** $\varepsilon_b = \bar{e}$ **.**

APPENDIX 3
Grants with compulsory return

We demonstrate here that, when distributing grants to students whose productivity is superior to $\bar{e}$, the allocation should begin with the brightest students (whose productivity is equal to e_2).
Let us assume that grants are given to students in the productivity interval $[\boldsymbol{\varepsilon}, \tilde{e}]$, with $\bar{e} \le \varepsilon \le \tilde{e} \le e_2$.
the corresponding welfare benefit is

$$W = \int_{\varepsilon}^{\tilde{e}} p(e)(e - I^*)de$$

The budget constraint is $S(\tilde{e} - \varepsilon) = kF$, and since $S = p(\tilde{e})(w^* - w_1)$, this constraint can be written $p(\tilde{e})(\tilde{e} - \varepsilon) = f$, which gives

$$\varepsilon = \tilde{e} - \frac{f}{p(\tilde{e})} = \tilde{e} - \frac{f(E_2 - E_1)}{\tilde{e} - E_1}$$

The derivative of W with respect to $\tilde{e}$ is

$$\frac{dW}{d\tilde{e}} = p(\tilde{e})(\tilde{e} - I^*) - \frac{d\varepsilon}{d\tilde{e}} p(\varepsilon)(\varepsilon - I^*)$$

with

$$\frac{d\varepsilon}{d\tilde{e}} = 1 + \frac{f(E_2 - E_1)}{(\tilde{e} - E_1)^2} = 1 + \frac{\tilde{e} - \varepsilon}{\tilde{e} - E_1}$$

$$\frac{dW}{d\tilde{e}} > 0 \;\leftrightarrow\; \frac{p(\tilde{e})(\tilde{e} - I^*)}{p(\varepsilon)(\varepsilon - I^*)} = \frac{(\tilde{e} - E_1)}{(\varepsilon - E_1)} \frac{(\tilde{e} - I^*)}{(\varepsilon - I^*)} > 1 + \frac{\tilde{e} - \varepsilon}{\tilde{e} - E_1}$$

By posing $\tilde{e} - \varepsilon = l$, $\frac{(\tilde{e} - E_1)}{(\varepsilon - E_1)} = 1 + \frac{l}{\varepsilon - E_1}$ and $\frac{(\tilde{e} - I^*)}{(\varepsilon - I^*)} = 1 + \frac{l}{\varepsilon - I^*}$
so that this condition may also be written

$$\left(1 + \frac{l}{\varepsilon - E_1}\right)\left(1 + \frac{l}{\varepsilon - I^*}\right) > 1 + \frac{l}{\tilde{e} - E_1}$$

But for $\le \tilde{e}$, $1 + \frac{l}{\varepsilon - E_1} \ge 1 + \frac{l}{\tilde{e} - E_1}$ and the condition is always met.
W is thus an increasing function of $\tilde{e}$, and the optimum corresponds to $\tilde{e} = e_2$.

APPENDIX 4
Welfare comparisons

4.1 Proof of Proposition 5

Let ΔW_b^* be the maximal value of W_b , when a grant is unconditional, which corresponds to the optimal values, $\bar{e}_b{}^*$and ε_b^*, of the integral limits $\bar{e}_b$ and ε_b. It will be shown that the couple $\bar{e}_{OR}$ = $\bar{e}_b{}^*$ and ε_{OR} = ε_b^* is financially attainable in the case of a grant scheme with an optional return. This couple results in the same welfare level, ΔW_b^*, as that which pertains to an unconditional grant. Consequently, the maximal attainable level of welfare in this case is superior, or equal, to ΔW_b^*.

The couple ($\bar{e}_b{}^*$, ε_b^*) satisfies naturally the budget constraint for the previously analyzed case where there is no conditionality, as specified by equation 13:

$$\text{(25.)}\ (\varepsilon_b^* - \bar{e}_b{}^*)(\bar{e} - \bar{e}_b{}^*) = kF$$

The objective then is to demonstrate that for the corresponding values in the case of optional return, $\bar{e}_{OR}$ and ε_{OR}, the budgetary constraint for such a scheme is not necessarily saturated, i.e.:

$$\frac{\bar{e} - \bar{e}_b{}^*}{E_2 - \bar{e}_b{}^*}\int_{\bar{e}_b{}^*}^{\varepsilon_b^*}(E_2 - e)de \leq kF$$

Combined with (25.), this inequality may also be expressed as:

$$\int_{\bar{e}_b{}^*}^{\varepsilon_b^*}(E_2 - e)de \leq (E_2 - e_b^*)\,(\varepsilon_b^* - \bar{e}_b{}^*)$$

These inequality conditions hold, since it is straightforward to demonstrate that the following equivalent inequality applies:

$$\int_{\bar{e}_b{}^*}^{\varepsilon_b^*} e\,de \geq \bar{e}_b{}^*(\varepsilon_b^* - \bar{e}_b{}^*).$$

4.2 Proof of Proposition 6

Part a)

An initial comparison is between the welfare effects of unconditional grants, as compared with those requiring a compulsory return. First, it will be

assumed that the latter grants are distributed only to individuals below the threshold $\bar{e}$. When the overall educational budget F is quite small, it follows that, since $\bar{e}_{CR}$ approaches $\bar{e}$, the change in welfare for a scheme with compulsory return can be approximated by: $\Delta W_{CR} = \frac{N}{e_2 - e_1}\int_{\bar{e}_{CR}}^{\bar{e}}(e - A)\, de \cong \frac{N}{e_2 - e_1}(\bar{e} - \bar{e}_{CR})(\bar{e} - A)$. In this instance the budget constraint, specified by equation 13, amounts to:

$$\bar{e} - \bar{e}_{CR} = \frac{e_2 - e_1}{N}\frac{F}{S_{CR}}$$

so that

$$\Delta W_{CR} \cong (\bar{e} - A)\frac{F}{S_{CR}} = \theta_1 F$$

In comparison unconditional grants are distributed on $[\bar{e}_b, \varepsilon_b]$, where $\varepsilon_b \leq \bar{e}$, so that the corresponding budget constraint is given by: $(\varepsilon_b - \bar{e_b})(\bar{e} - \bar{e_b}) = kF$. When F is small, ε_b and $\bar{e_b}$ are near to $\bar{e}$, so that $(\bar{e} - \bar{e_b})^2 \cong kF$.
Accordingly, the incremental change in welfare is:

$$\Delta W_B = \frac{\mathrm{N}}{\mathrm{e}_2 - \mathrm{e}_1}\int_{\overline{e_b}}^{\varepsilon_b} \varphi(\mathrm{e})\, \mathrm{de} \cong \frac{\mathrm{N}}{\mathrm{e}_2 - \mathrm{e}_1}(\bar{e} - \overline{e_b})\varphi(\bar{e}) = \theta_2\sqrt{F}$$

By then comparing $\Delta W_{CR}$and ΔW_B, it can easily be deduced that for $F < \left(\frac{\theta_2}{\theta_1}\right)^2$, a scheme with unconditional grants enhances welfare more than one entailing a compulsory return, so that $\Delta W_B > \Delta W_{CR}$.

Second, for the case where the overall budget, F, is again limited, but the conditional grants are distributed in the neighborhood of e_2, the budget constraint, provided in equation 30, requires that $e_2 - \varepsilon_2$ is proportional to F. The associated welfare change, $\Delta W'_{CR}$, is also approximatively proportional to $e_2 - \varepsilon_2$ and, consequently, also to F. Hence, the demonstration is identical to the previous case. Finally, in light of proposition 5, grants with optional returns dominate unconditional grants, so that they are also superior to grants with compulsory return.

Part b)

The demonstration for this scenario where public educational funding is relatively unconstrained is rather trivial. In such a scenario grants with compulsory return will be offered to as many of the most capable students, as funding will allow, and will then actually be accepted, when the subsidies for those individuals are sufficiently large. In contrast, for the unconditional and optional return systems, grants will only be allocated to students whose abilities are inferior or equal to the threshold $\bar{e}$.